Laker's Linear Learnings

MIT 18.700
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In the autumn of 2018, I took MIT's 18.700 Theoretical Linear Algebra course through OCW, as offered in 2013. Every week, I completed the assigned problem set then met with Khan Academy engineer and MIT alumnus Ben Kraft. Ben offered advice on how I could improve my solutions; I have immortalized it here.

Problem Set 1

1. When necessary, prove the uniqueness of a result.
2. Prove any assumption that is not immediately obvious.
3. Avoid words like "clearly" and "obviously." Instead, write what makes the claim clear or obvious.
4. Whenever possible, build and apply intuition about the statement being proven.

Problem Set 2

1. Only combine equalities to imply a relation.
2. Add text or a line break after lemmas for readability.
3. Only write "consider $v \in V$" or something similar if v is intended to be an arbitrary element of V.
4. Strive for concision in the wordy parts of proofs.
5. Include all of a detail or none of it to avoid confusion.
6. Always consider the optimal place for a sentence.
7. Always consider alternative methods of proof.

Problem Set 3

1. Expand as little as possible when dealing with abstract ideas.
2. Ask, "What prevents this from being true?"
3. Always consider the best way to attack a problem.
4. Write a synopsis before detailed proofs.
5. Make free use of newline math for readability.
6. Avoid starting sentences with math. Instead, write "The equation ..." or something similar.
7. When possible, combine two directions into a chain of if and only ifs.

Problem Set 4

1. Describe all algorithms used, even if they are simple.
2. Total options is the product of each individual option if the items are sequentially chosen.
3. If necessary, mention when a result is invariant of a variable.
4. State the basis before the matrix unless it is the standard basis.
5. Avoid pretentious wording: directional hypothesis $\longrightarrow$ our assumption.
6. Ask, "Am I making assumptions?"
7. Say, "eigenvectors look like [visual intuition]" when it's not obvious.
8. Wrap up and compile the solution at the end when they are otherwise scattered.
9. Write equations such that the first and final quantities align to what will be concluded.

Problem Set 5

1. When applicable, break up a matrix into its columns then analyze each separately.
2. Justify all statements that are not exceedingly obvious.

Problem Set 6

1. Rename different variables that share the same name.
2. State what "we wish to show."
3. Note temporary constructions to clarify their purpose.
4. Attempt to generalize expressions that hint at a pattern.

Problem Set 7

1. Scan drafts for typos.
2. Take care to not double-count.
3. Think hard about the number theory, then take a wild stab.
4. Never place an object from one structure into another unless justified.
5. Be wary of subtle differences that separate similar structures.
6. For specific constructions, state "We will construct * such that ..." instead of "Let * be such that ...."

Problem Set 8

1. Minimize writing everything out in favor of using properties of the objects.
2. Strive for cleaner proofs.
3. Factor common numbers out of matrices to improve readability.

Problem Set 9

1. When proving axioms, remember to prove them all.
2. Certain eigenvalues are unsolvable, because the characteristic equation has degree five or higher.
3. Seek the best LaTeX command for what needs to be typed.

Life Lessons

1. Create a system to ensure that arriving late to meetings won't happen.
2. Keep track of what you don't understand, but don't wait until it clicks to progress.
3. Often, a shallow bridge of understanding is necessary to reach deeper results down the tunnel that then loop back to motivate the current material and reveal its essence.